Millikin University
Student Learning in Quantitative Reasoning

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Executive Summary: Quantitative Reasoning (QR) is a non-sequential requirement in the MPSL. All Millikin students are required to fulfill this by taking a designated QR course. A goal of this program is that through quality advising, students take a QR course that fits the needs of the student's academic area. Here we report a full year (**fall of 2012 and spring 2013**) of data collection and analysis for purposes of assessment. We find that QR courses in the traditional program are meeting our goals at the green level.

Goals
A student who successfully completes a Millikin QR course will demonstrate the ability to:

(1) use deductive reasoning in a formal, symbolic, axiomatic system, and
(2) apply the theorems of the system to solve appropriate problems.

The learning goals of the quantitative reasoning requirement are part of broader aims of this requirement. Through this requirement Millikin hopes to:

(a) To offer the basic quantitative reasoning skills necessary for success in every profession. All work involves understanding the basics of numerical, statistical, or logical analysis. This type of thinking is fundamental to understanding the world and no career is exempt from this way of knowing.
(b) To prepare students to be competent citizens by developing the quantitative skills necessary to understand fundamental reasoning that involves numbers, statistics, or logical reasoning. Citizens must be able to understand e.g., graphs, detect faulty statistical analysis, or spot basic flaws in reasoning. These courses serve democracy by developing such skills.

Clearly these two goals support the first two university-wide goals:

University Goal 1: Millikin students will prepare for professional success.

> University Goal 2: Millikin students will actively engage in the responsibilities of citizenship in their communities.

Each career has specific requirements in regard to deductive reasoning. A psychologist may work on a daily basis with statistical analysis on her experimental data, while a lawyer might need to spot an *ad hominem* fallacy in an oral argument. A manager will need to understand what the accountant means, while an artist might need to prepare a usable table to demonstrate the efficiency of their program in a grant request. All of these are examples of quantitative reasoning. The broad range of careers benefits from the range of courses offered that teach quantitative skills. Many humanities, arts, and pre-law students take logic. Business and nursing require math courses. Social scientists need statistics. Careful advising that includes career goals helps students select the quantitative course that best supports their individual aim, while all of these courses prepare students for success in their profession.

In addition to helping students flourish in their job, quantitative reasoning is basic to good citizenship. To be a fully functioning citizen in this country, you must be able to understanding the news and ask appropriate questions. Those tasks require quantitative reasoning. One cannot pick up a newspaper without finding statistics about the amount of oil produced in Iraq, the population growth, the rate of immigration, and so on. Unless you can understand the basics of these figures, charts, graphs, and the reasoning they are based on, you cannot sort out the bogus from the noteworthy. At its most basic, this requirement aims to produce students who can become these kind of citizens.

The learning goals are simple: learn to use deducting reasoning and be able to apply it appropriately. The larger goals are less concrete: to give students the quantitative skills they need to succeed as a citizen and on the job.

Snapshot

<u>Faculty</u>

The faculty who taught the QR courses for Fall 2012 and Spring 2013 are listed in Tables 1 and 2. All traditional courses were taught by full-time faculty with PhD's.

<u>Facilities</u>

None of the courses require facilities beyond classrooms. Most require technology rooms with computers and projectors.

<u>Types and number of students served</u>

All students are required to complete a QR course.

<u>Number and types of courses taught</u>

Select courses in Mathematics, Philosophy, and Behavioral Sciences meet the QR requirement. The number and types of courses taught are detailed in Table 1. All of these courses were chosen because they fulfill the requirement of manipulation of symbols according to rigorous inference rules or algorithms.

Course Number	Title	Faculty
MA109	Finite Mathematics	Joe Stickles
MA110	College Algebra	Dan Miller Paula Stickles
MA115	Trigonometry	Randal Beck
MA120	Elementary Probability and Statistics	Eun-Joo Lee Randal Beck
MA 140	Calculus I	James Rauff
PS201/SO201	Statistical Methods in the Behavioral Sciences	Linda Collinsworth
PH213	Logic	Michael Hartsock
IN207	Honors Seminar in Mathematics	James Rauff

Table 1. Fall 2011 Courses meeting the QR requirement

Course Number	Title	Faculty
MA109	Finite Mathematics	Eun-Joo Lee
MA110	College Algebra	Paula Stickles Dan Miller
MA 114	Trigonometry	Joe Stickles
MA120	Elementary Probability and Statistics	Eun-Joo Lee Randal Beck
MA140	Calculus I	Paula Stickles
PS201/SO201	Statistical Methods in the Behavioral Sciences	Linda Collinsworth
MA 208	Discrete Mathematics	Joe Stickles
IN207	Honors Seminar in Mathematics	James Rauff

Table 2. Spring 2013 Courses meeting the QR requirement

Programs

The QR courses serve all programs of the university. Some programs have designated courses to meet the requirement. Majors in the Behavioral Sciences (Human Services, Sociology, Psychology) and Nursing meet the requirement with PS021/SO201. Majors in Philosophy meet the requirement with PH213. Tabor School of Business students meet the requirement with MA120. Many Natural Science and Mathematics majors meet the requirement with either MA114 or MA140, while many Early Childhood and Elementary Education majors meet the requirement with either MA112 or MA125.

Partnerships

There are no external partnerships involving the Quantitative Reasoning requirement.

The Learning Story

Most of Millikin's programs of study have particular QR needs. The QR course provides the kinds of experiences in symbolic, deductive reasoning required for professional success in these areas. Examples of QR topics in action include statistics in nursing, business and the behavioral sciences, calculus in the natural and physical sciences, and logic in the humanities. In addition, students of the arts gain experiences in symbolic, deductive reasoning that will enhance their ability to grow as educated democratic citizens (e.g., financial mathematics, voting models, and resource analysis).

The particular nature of the experience varies from student to student according to their educational, professional, and personal goals. Not every student needs to learn calculus or statistics or symbolic logic. Nevertheless, every Millikin student will have an experience that challenges him or her to apply rigorous deductive reasoning. The QR requirement provides this experience in the context of the student's major field or in the context of the student's participation in a democratic society.

The QR requirement is seen in action when a biology student successfully applies the integral and differential calculus in modeling the growth of a population of field mice, when a business student successfully argues for the marketability of a new product based upon a sound statistical analysis of a survey, when a pre-law student successfully answers a LSAT question involving syllogisms, or when a musical theatre major successfully obtains a loan for a new car after knowledgably negotiating rates and terms.

Assessment Methods.

The QR goal is a meta-goal encompassing a wide variety of fields and techniques that falls under the broad heading of formal, symbolic axiomatic systems. Hence, the assessment method is equally wide and encompassing. The key to success is the content of the QR courses. Does the course provide the student with experiences in symbolic deductive reasoning?

<u>One Program Review Processes</u>

Syllabi review. Usually, a quick examination of the syllabus is sufficient.

One Assessment of Student Performance

Samples of final exams will be evaluated, rating how well students demonstrate the ability to use deductive reasoning in a formal, symbolic, axiomatic system, and to apply the theorems of the system to solve appropriate problems.

Assessment Data

The following data were collected by the QR coordinator.

1. Syllabi for all traditional QR courses and some PACE courses.
2. Sample problems were taken from final exams for almost all traditional QR courses and some PACE courses. Instructors of these courses were asked to identify one problem on their exam that could be used to assess the deductive reasoning learning goal and to identify one problem on their exam that could be used to assess the theorem application goal. The instructors then randomly selected five students' final exams and assessed each student's work according to the assessment rubric the Quantitative Reasoning Task Force developed (see appendix). The assessments are summarized in the tables below.

Fall 2012			
Course	**Good**	**Average**	**Poor**
MA 109 01	3	1	1
MA 109 02	4	0	1
MA 110 01	No Data Available		
MA 110 02	4	0	1
MA 110 03	1	2	2
MA 115 01	3	2	0
MA 120 02	1	1	3
MA 120 03	No Data Available		
MA 120 04	No Data Available		
MA 140 01	3	1	1
PS 201 01	No Data Available		
IN 207 01	4	0	1
PH 213 01	3	1	1
Totals	26 58%	8 18%	11 24%

Table 3. Deductive Reasoning Fall 2012

Spring 2013			
Course	**Good**	**Average**	**Poor**
MA109 01	No Data Available		
MA109 02	No Data Available		
MA110 01	2	0	3
MA110 02	1	2	2
MA110 03	No Data Available		
MA115 01	No Data Available		
MA120 01	2	2	1
MA120 02	No Data Available		
MA120 03	3	1	1
MA140 01	3	0	2
PS201 01	2	1	2
MA208 01	No Data Available		
Totals	13 43%	6 20%	11 37%

Table 4. Deductive Reasoning Spring 2013

Fall 2012			
Course	**Good**	**Average**	**Poor**
MA 109 01	5	0	0
MA 109 02	5	0	0
MA 110 01	No Data Available		
MA 110 02	4	1	0
MA 110 03	1	3	1
MA 115 01	5	0	0
MA 120 02	3	2	0
MA 120 03	No Data Available		
MA 120 04	No Data Available		
MA 140 01	1	2	2
PS 201 01	No Data Available		
IN 207 01	2	3	0
PH 213 01	2	3	0
Totals	28 62%	14 31%	3 7%

Table 5. Theorem Application Fall 2012

Spring 2013			
Course	**Good**	**Average**	**Poor**
MA109 01	No Data Available		
MA109 02	No Data Available		
MA110 01	5	0	0
MA110 02	4	0	1
MA110 03	No Data Available		
MA115 01	No Data Available		
MA120 01	3	2	0
MA120 02	No Data Available		
MA120 03	1	3	1
MA140 01	1	1	3
PS201 01	3	2	0
MA208 01	No Data Available		
Totals	17 57%	8 27%	5 17%

Table 6. Theorem Application Spring 2012

After piloting the QR rubric in Spring 2007, we found that all instructors provided the necessary data without asking a single question about the selection process or about the rubric. This has continued for the twelve semesters of data collection done for 2007-08, 2008-09, 2009-10, 2010-11, 2011-12 and 2012-13. Therefore, we conclude that the QR assessment instrument is easy to implement and will not make any changes to the procedure for 2012-13. We will continue to evaluate the data collection process as we move forward with the assessments in future years.

Analysis of Assessment Results

1. The review of the syllabi for all QR courses revealed that almost all of the QR syllabi that were given to the QR director from 2012-13 explicitly stated the QR learning goals and all of them included topics that directly reflected these goals.

2. The evaluation of random samples of completed final exams concluded the following status of student performance with respect to the QR student learning outcomes.

(1) use deductive reasoning in a formal, symbolic, axiomatic system.

Seventy-six percent for Fall 2012 and 63% for Spring 2013 of the assessed student completed finals were rated good or average by the instructors.

(2) apply the theorems of the system to solve appropriate problems.

Ninety-three percent for Fall 2012 and 83% for Spring 2013 of the assessed student completed finals were rated good or average by the instructors.

Improvement Plans

We are pleased that the data indicate that the QR goals are being met. Nevertheless, we see room for improvement in several areas.

(1) Almost all QR syllabi contain the QR learning goals explicity. While this is very impressive, our long-term goal is to have 100% of QR syllabi include the explicit QR learning goals. To help achieve this goal, before each semester begins, the QR coordinator sends an email to QR instructors reminding them to include the goals in their syllabi. (The goals themselves are included in these emails.)
(2) The theorem application goal is being met. The number of “poor” ratings has ended its downward trend last year. It went from 29% in Spring 2007, to 21.4% for the 2007-08 academic year, to 17.1% for the 2008-09 academic year, to 15.6% for the 2009-10 academic year. The percentage of poor responses for theorem application went up to 20.6% in 2010-11. However, the percentage dropped down to 17.5% in 2011-12. This is consistent with the percentages over the past 4 years.
(3) The percentages of average and good responses have dropped during the 2012-2013 academic year. The percentages for the deductive reasoning goal will be watched during the next year to determine if this normal data variation or the beginning of a trend.
(4) Collecting information from QR courses taught in the traditional program needs to improve. Instructors in the traditional program need to do a better job of reporting their results to the QR coordinator and the QR coordinator needs to do a better job of following up on instructor’s who have not submitted their reports.
(5) Collecting information from QR courses taught in the PACE program continues to be a challenge. We need to work with the PACE office in order to get vital assessment data from these courses.

Appendix.

Rubric for Assessing Student Achievement of Quantitative Reasoning Learning Outcome Goals

	Good	Average	Poor
Deductive Reasoning	• Student uses proper symbolic notation in the context of the stated problem • Student manipulates these symbols according to the rules of the axiomatic system • Student achieves desired directive of the problem with no (or a few minor) errors	• Student work achieves exactly two of the following: 1. Student uses proper symbolic notation in the context of the stated problem 2. Student manipulates these symbols according to the rules of the axiomatic system 3. Student achieves desired directive of the problem with no (or a few minor) errors	• Student work achieves no more than one of the following: 1. Student uses proper symbolic notation in the context of the stated problem 2. Student manipulates these symbols according to the rules of the axiomatic system 3. Student achieves desired directive of the problem with no (or a few minor) errors
Theorem Application	• Student correctly selects theorem(s) necessary to solve stated problem • Student performs all necessary calculations needed to apply theorem with no (or a few minor) errors • Student uses selected theorem(s) to form a correct conclusion on the basis of the computations	• Student correctly selects theorem(s) necessary to solve stated problem • Student's work falls into one of the following two categories: 1. Student made major computational errors, but made a correct conclusion based on the computations 2. Student made no (or a few minor) errors in computations, but made an incorrect conclusion made on the computations	• Student work falls into one of the following two categories: 1. Student did not correctly select theorem(s) necessary to solve stated problem 2. Student did correctly select theorem(s) necessary to solve, but made major computational errors and made an incorrect conclusion based on the computations

Examples

Deductive Reasoning

MA 120 students are asked to solve the following problem: "On the 2006 SAT Mathematics Exam, the average score was 518 with a standard deviation of 115. Assuming these scores are normally distributed, what percentage of students scored higher than 596 on the 2006 SAT Mathematics Exam?"

This problem requires students to select the *z*-score formula, substitute the given information into the formula, perform the *z*-score computation, find the corresponding percentage in a normal distribution table, and manipulate the number from the table to answer the given question correctly.

The selection of the *z*-score formula and correct substitution into the formula would constitute using proper symbolic notation in the context of the problem because students must extract the necessary information from the problem and replace the unknown quantities in formula with this information. Correctly computing the *z*-score and finding the correct corresponding percentage in the normal distribution table would constitute manipulating these symbols according to the rules of the axiomatic system. Finally, correctly extracting the percentage from the table to answer the given question shows the student not only can follow the rules of the system, but also can reach a valid conclusion after applying these rules, thereby achieving the desired directive.

Good response: *The z-score for an SAT Mathematics score of 596 is*

$$z = \frac{596 - 518}{115} = \frac{78}{115} \approx 0.68$$

The percentage corresponding to a z-score of 0.68 in the normal distribution table is 25.17%. This is the percentage that lies between the data point and the mean, or in this case, the percentage of students who scored between 518 and 596. So, the percentage of students who scored higher than 596 on the 2006 SAT Mathematics Exam is 50% - 25.17% = 24.83%

Average response: *The z-score for an SAT Mathematics score of 596 is*

$$z = \frac{596 - 518}{115}$$
$$= \frac{78}{115}$$
$$\approx 0.68$$

The percentage corresponding to a z-score of 0.68 in the normal distribution table is 25.17%. So, the percentage of students who scored higher than 596 on the 2006 SAT Mathematics Exam is 25.17%.

This student response demonstrates the first two traits of a good response, but not the third. The student has correctly used proper symbolic notation in the context of the stated problem by selecting the *z*-score formula and inserting the numbers into the formula correctly, and manipulated these symbols according to the rules of the axiomatic system by finding the correct *z*-score and corresponding percentage from the normal distribution table. However, the student has simply used the percentage found in the table as the final answer and thus has not achieved the desired directive of the problem.

Poor response: *The z-score for an SAT Mathematics score of 596 is*

$$z = \frac{596 - 518}{115}$$
$$= \frac{78}{115}$$
$$\approx 0.68$$

So, the percentage of students who scored higher than 596 on the 2006 SAT Mathematics Exam is 68%.

This student response demonstrates the first trait of a good response, but not the other two. The student has correctly selected the *z*-score formula and substitutes the given information correctly. However, the student did not apply the rules of the axiomatic system correctly since the student did not find the corresponding percentage in the normal distribution table. Also, the student did not achieve the desired directive because the student did not apply the rules of the system correctly.

<u>Deductive Reasoning</u>

PH 213 students are asked the following question: “Using a truth table, determine whether the statements $\neg p \vee q$ and $p \rightarrow q$ are equivalent.”

This problem requires students to construct a truth table for each of the statements and determine if the statements have identical final columns in their truth tables.

A student uses proper symbolic notation in the context of the stated problem when the student sets up the truth table correctly be creating the appropriate columns for the statements. When the student fills out the truth table correctly, that student has manipulated these symbols according to the rules of the axiomatic system. Finally, the desired directive is achieved when the student compares the final column of each truth table to determine whether or not the statements are equivalent.

Good response:

The truth table for $\neg p \vee q$ *is*

p	q	$\neg p$	$\neg p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

The truth table for $p \rightarrow q$ *is*

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Since the last columns of the truth tables are identical, the statements are equivalent.

The student set up the truth tables correctly, including having an extra column in the first table for $\neg p$ to use as an intermediate step. So, this student has used proper symbolic notation for this problem. This student has also correctly filled in the truth tables, demonstrating the ability to use the rules of the axiomatic system in this context. Finally, the student correctly assesses the statements are in fact equivalent by examining the final columns of the truth table, thereby achieving the desired directive of the problem.

Average response:

The truth table for $\neg p \vee q$ *is*

p	q	$\neg p$	$\neg p \vee q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

The truth table for $p \to q$ *is*

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Since the last columns of the truth tables are not identical, the statements are not equivalent.

The student set up the truth tables correctly, including having an extra column in the first table for $\neg p$ to use as an intermediate step. So, this student has used proper symbolic notation for this problem. This student has not correctly filled in the truth tables; indeed, this student has confused the logical "or" with the logical "and". So, this student has not followed the rules of the axiomatic system correctly. However, the student does use their information to assess correctly (according to their computations) that the statements are not equivalent by examining the final columns of the truth table, thereby achieving the desired directive of the problem. Since this student work exhibits two of the three criteria of a good response, this response is rated as average.

Poor response:

The truth table for $\neg p \vee q$ *is*

p	q	$\neg p \vee q$
T	T	T
F	F	F

The truth table for $p \to q$ *is*

p	q	$p \to q$
T	T	T
F	F	T

Since the last columns of the truth tables are not identical, the statements are not equivalent.

The student has not set up the truth tables correctly because the student has eliminated to cases from consideration. So, this student has not used proper symbolic notation for this problem. This student also has not correctly filled in the truth tables, and with the missing rows in the truth tables, it is impossible to determine whether the student has confused the logical "or" with the logical "and", or if the student just randomly inserted some symbols. In either case, this student has not followed the rules of the axiomatic system correctly. However, the student does use their information to assess correctly (according to their computations) that the statements are not equivalent by examining the final columns of the truth table, thereby achieving the desired directive of the problem. Since this student work exhibits one of the three criteria of a good response, this response is rated as poor.

<u>Theorem Application</u>

MA 140 students are asked the following question: "For the function $f(x) = x^3 - 6x^2$, find the *x*-coordinates of the points at which *f(x)* has relative maxima and relative minima."

This problem requires students to find the derivative of *f(x)*, denoted $f'(x)$, to solve the equation $f'(x) = 0$ and to determine when $f'(x)$ is undefined to identify the critical numbers of *f(x)*, and to apply either the First Derivative Test or the Second Derivative Test to each of the critical numbers in order to identify which, if any, of the critical numbers correspond to relative maxima and which, if any, correspond to relative minima.

The computation of the derivative and solving $f'(x) = 0$ would qualify as performing all necessary calculations needed to apply the theorem. Students can show they correctly select the theorem necessary to solve stated problem by applying either the First Derivative Test or the Second Derivative Test. If students apply the theorem correctly, they will correctly identify where the maxima and minima are and thus demonstrate the third criterion of the rubric.

Good response:

$$f(x) = x^3 - 6x^2$$

$$f'(x) = 3x^2 - 12x$$

We see that $f'(x)$ *is undefined nowhere and that* $f'(x) = 0$ *when*

$$3x^2 - 12x = 0$$

$$3x(x-4) = 0$$

$$x = 0 \text{ or } x = 4$$

Now, $f''(x) = 6x - 12$. *Since* $f''(0) = -12 < 0$, *the Second Derivative Test tells us that* $x = 0$ *is the x-coordinate of a relative maximum of f(x). Since* $f''(4) = 12 > 0$, *the Second Derivative Test tells us that* $x = 4$ *is the x-coordinate of a relative minimum of f(x). So, the function* $f(x) = x^3 - 6x^2$ *has a relative maximum at* $x = 0$ *and a relative minimum at* $x = 4$.

This student response correctly computes the derivative and finds all the critical number. So, the second trait of a good response has been met. The first trait, the correct selection of the theorem, is shown by the student correctly selecting the Second Derivative Test to use. Finally, the student applies the theorem correctly by plugging the critical numbers into the second derivative and makes the correct conclusions based on these computations. Thus, the third trait of the good response has been met.

Average response:

$$f(x) = x^3 - 6x^2$$
$$f'(x) = 3x^2 - 12x$$

We see that $f'(x)$ *is undefined nowhere and that* $f'(x) = 0$ *when*

$$3x^2 - 12x = 0$$
$$3x(x-4) = 0$$
$$x = 0 \text{ or } x = 4$$

Now, $f''(x) = 6x - 12$. *Since* $f''(0) = -12 < 0$*, the Second Derivative Test tells us that* $x = 0$ *is the x-coordinate of a relative minimum of f(x). Since* $f''(4) = 12 > 0$*, the Second Derivative Test tells us that* $x = 4$ *is the x-coordinate of a relative maximum of f(x). So, the function* $f(x) = x^3 - 6x^2$ *has a relative minimum at* $x = 0$ *and a relative maximum at* $x = 4$.

This student response correctly computes the derivative and finds all the critical number. So, the second trait of a good response has been met. The first trait, the correct selection of the theorem, is shown by the student correctly selecting the Second Derivative Test to use. The student applies the theorem correctly by plugging the critical numbers into the second derivative; however, the student makes incorrect conclusions by saying the maximum is a minimum, and vice versa. Thus, the third trait of the good response has not been met.

Poor response:

$$f(x) = x^3 - 6x^2$$
$$f'(x) = 3x^2 - 12x$$

We see that $f'(x)$ *is undefined nowhere and that* $f'(x) = 0$ *when*

$$3x^2 - 12x = 0$$
$$3x(x-4) = 0$$
$$x = 0 \text{ or } x = 4$$

Now, $f(0) = 0$ *and* $f(4) = -32$. *So, f(x) has a relative minimum at* $x = 4$ *and a relative maximum at* $x = 0$.

This student has performed correct calculations when finding the derivative and the critical numbers. However, the student has not applied the appropriate theorem to arrive at a correct conclusion. In fact, this student has attempted to use the Extreme Value Theorem, which deals with the *absolute* maximum and *absolute* minimum of a function under certain conditions. This is a major conceptual error and thus warrants a rating of poor.