Millikin University
Student Learning in Quantitative Reasoning

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Executive Summary: Quantitative Reasoning (QR) is a non-sequential requirement in the MPSL. All Millikin students are required to fulfill this by taking a designated QR course. A goal of this program is that through quality advising, students take a QR course that fits the needs of the student's academic area. Here we report on the first semester of data collection and analysis for purposes of assessment. We find that QR courses in the traditional program are meeting our goals at the green level.

Goals

A student who successfully completes a Millikin QR course will demonstrate the ability to:

(1) use deductive reasoning in a formal, symbolic, axiomatic system, and
(2) apply the theorems of the system to solve appropriate problems.

The learning goals of the quantitative reasoning requirement are part of broader aims of this requirement. Through this requirement Millikin hopes to:

(a) To offer the basic quantitative reasoning skills necessary for success in every profession. All work involves understanding the basics of numerical, statistical, or logical analysis. This type of thinking is fundamental to understanding the world and no career is exempt from this way of knowing.
(b) To prepare students to be competent citizens by developing the quantitative skills necessary to understand fundamental reasoning that involves numbers, statistics, or logical reasoning. Citizens must be able to understand e.g., graphs, detect faulty statistical analysis, or spot basic flaws in reasoning. These courses serve democracy by developing such skills.

Clearly these two goals support the first two university-wide goals:

> University Goal 1: Millikin students will prepare for professional success.
> University Goal 2: Millikin students will actively engage in the responsibilities of citizenship in their communities.

Each career has specific requirements in regard to deductive reasoning. A psychologist may work on a daily basis with statistical analysis on her experimental data, while a lawyer might need to spot an *ad hominem* fallacy in an oral argument. A manager will need to understand what the accountant means, while an artist might need to prepare a usable table to demonstrate the efficiency of their program in a grant request. All of these are examples of quantitative reasoning. The broad range of careers benefits from the range of courses offered that teach quantitative skills. Many humanities, arts, and pre-law students take logic. Business and nursing require math courses. Social scientists need statistics. Careful advising that includes career goals helps students select the quantitative course that best supports their individual aim, while all of these courses prepare students for success in their profession.

In addition to helping students flourish in their job, quantitative reasoning is basic to good citizenship. To be a fully functioning citizen in this country, you must be able to understanding the news and ask appropriate questions. Those tasks require quantitative reasoning. One cannot pick up a newspaper without finding statistics about the amount of oil produced in Iraq, the population growth, the rate of immigration, and so on. Unless you can understand the basics of these figures, charts, graphs, and the reasoning they are based on, you cannot sort out the bogus from the noteworthy. At its most basic, this requirement aims to produce students who can become these kind of citizens.

The learning goals are simple: learn to use deducting reasoning and be able to apply it appropriately. The larger goals are less concrete: to give students the quantitative skills they need to succeed as a citizen and on the job.

Snapshot

<u>Faculty</u>

The faculty currently teaching the QR courses are listed in Table 1. All are full-time faculty with PhD's except Carol Sudduth, an adjunct with an MA in Mathematics.

Facilities

None of the courses require facilities beyond classrooms. Most require technology rooms with computers and projectors.

Types and number of students served

All students are required to complete a QR course.

Number and types of courses taught

Select courses in Mathematics, Philosophy, and Behavioral Sciences meet the QR requirement. The number and types of courses taught are detailed in Table 1. All of these courses were chosen because they fulfill the requirement of manipulation of symbols according to rigorous inference rules or algorithms.

Table 1. Courses meeting the QR requirement

Course Number	Title	Faculty	Notes
MA 112	Mathematical Content for Elementary School Teachers	Paula Stickles	
MA 114	Functions	Daniel Miller	
MA 117	Finite Mathematics	Carol Sudduth	
MA 120	Elementary Probability and Statistics	Michael Fearheiley Eun-Joo Lee Randal Beck	
MA 125	Mathematics and the World	Joe Stickles Carol Sudduth	
MA 140	Calculus	Eun-Joo Lee Joe Stickles	
MA 208	Discrete Mathematics	James Rauff Michael Rodgers	
PH 213	Critical Thinking: Logic	Jo Ellen Jacobs	Spring only
PS201/SO201	Statistical Methods in the Behavioral Sciences	Linda Collinsworth	
IN 207	Honors Seminar in Mathematics	James Rauff	Honors Program students only

<u>Programs</u>

The QR courses serve all programs of the university. Some programs have designated courses to meet the requirement. Majors in the Behavioral Sciences (Human Services, Sociology, Psychology) and Nursing meet the requirement with PS021/SO201. Majors in Philosophy meet the requirement with PH213. Tabor School of Business students meet the requirement with MA120.

<u>Partnerships</u>

There are no external partnerships involving the Quantitative Reasoning requirement.

The Learning Story

Most of Millikin's programs of study have particular QR needs. The QR course provides the kinds of experiences in symbolic, deductive reasoning required for professional success in these areas. Examples of QR topics in action include statistics in nursing, business and the behavioral sciences, calculus in the natural and physical sciences, and logic in the humanities. In addition, students of the arts gain experiences in symbolic, deductive reasoning that will enhance their ability to grow as educated democratic citizens (e.g., financial mathematics, voting models, and resource analysis).

The particular nature of the experience varies from student to student according to their educational, professional, and personal goals. Not every student needs to learn calculus or statistics or symbolic logic. Nevertheless, every Millikin student will have an experience that challenges him or her to apply rigorous deductive reasoning. The QR requirement provides this experience in the context of the student's major field or in the context of the student's participation in a democratic society.

The QR requirement is seen in action when a biology student successfully applies the integral and differential calculus in modeling the growth of a population of field mice, when a business student successfully argues for the marketability of a new product based upon a sound statistical analysis of a survey, when a pre-law student successfully answers a LSAT question involving syllogisms, or when a musical theatre major successfully obtains a loan for a new car after knowledgably negotiating rates and terms.

Assessment Methods.

The QR goal is a meta-goal encompassing a wide variety of fields and techniques that falls under the broad heading of formal, symbolic axiomatic systems. Hence, the assessment method is equally wide and encompassing. The key to success is the content of the QR courses. Does the course provide the student with experiences in symbolic deductive reasoning?

<u>Two Program Review Processes</u>

(1) Syllabi review. Usually, a quick examination of the syllabus is sufficient.
(2) Review what QR courses are being offered that academic year, which ones students are taking, and whether or not the courses they are taking are the best for their majors. These reviews will be used to help advisors understand the connections between QR courses and their areas.

<u>One Assessment of Student Performance</u>

Samples of final exams will be evaluated, rating how well students demonstrate the ability to use deductive reasoning in a formal, symbolic, axiomatic system, and to apply the theorems of the system to solve appropriate problems.

Assessment Data

The following data were collected by the QR task force.

(1) The QR assessment team collected and reviewed the syllabi for all QR courses offered in Spring 2007. Final exams for some of these courses were collected, but not a large enough percentage were obtained to be used for assessment.
(2) The QR assessment team collected data about QR course offerings and student enrollment trends (which courses are students taking from various disciplines) for the academic year 2006-2007. These data are presented in the table below. In this table, the column headings are to be understood as follows:

Bus - all majors in the Tabor School of Business
Ed. – all majors in the School of Education
Sci. – all majors in the NSM Division of CAS
Nur. – all majors in the School of Nursing
Art. – all majors in the College of Fine Arts
Beh. – all majors in the Behavioral Science Division of CAS

Table 2. Enrollment Trends 2006-2007 (Traditional Program)

Course	Bus.	Ed.	Sci.	Nur.	Art.	Beh.	Other	*Total*
MA112	1	24	0	0	3	1	2	*31*
MA114	4	3	32	0	2	0	8	*49*
MA120	74	23	19	18	22	8	34	*198*
MA125	12	20	0	1	46	13	30	*122*
MA140	6	11	57	0	3	1	10	*88*
MA208	5	7	19	0	0	0	0	*31*
PH213	0	0	2	0	7	0	11	*20*
PS201	1	0	18	26	17	18	30	*110*
SO201	1	1	12	2	5	12	7	*40*
Total	*104*	*89*	*159*	*47*	*105*	*53*	*132*	*689*

Gold cells indicate that this major category supplied more than 37% of students enrolled in the course.

(3) The QR assessment team collected student data from nine quantitative reasoning courses offered in Spring 2007. Data from three courses, MA 120 02, MA 120 03, and MA 120 04, were not collected because Dr. Lee left the country for a conference before we could get data from her sections. We asked the instructors of these courses to identify one problem on their exam that could be used to assess the deductive reasoning learning goal and to identify one problem on their exam that could be used to assess the theorem application goal. The instructors then randomly selected five students' final exams and assessed each student's work according to the assessment rubric the Quantitative Reasoning Task Force developed (see appendix). The assessments are summarized in the tables below.

Table 3. Deductive Reasoning

Course	Good	Average	Poor
MA 112 01	4	0	1
MA 114 01*	4	0	1
MA 120 01	2	2	1
MA 125 02	5	0	0
MA 125 03	4	0	1
MA 140 01	2	2	1
PH 213 01	3	2	0
PS 201 01*	5	0	0
PS 201 02*	5	0	0
Totals	34 $(75.\bar{5}\%)$	6 $(13.\bar{3}\%)$	5 $(11.\bar{1}\%)$

Table 4. Theorem Application

Course	Good	Average	Poor
MA 112 01	2	0	3
MA 114 01*	4	0	1
MA 120 01	0	2	3
MA 125 02	4	0	1
MA 125 03	2	2	1
MA 140 01	3	1	1
PH 213 01	3	2	0
PS 201 01*	3	0	2
PS 201 02*	4	0	1
Totals	25 $\left(55.\overline{5}\%\right)$	7 $\left(15.\overline{5}\%\right)$	13 $\left(28.\overline{8}\%\right)$

The asterisks next to course numbers indicate that the final exams for these courses were multiple choice exams. The only assessment that could be done was whether or not students were able to answer these questions correctly. So, a correct answer was assessed as “good,” and an incorrect answer was assessed as “poor.”

Since this was the first time we used the rubric to assess QR learning goals, we decided that we wanted to use the spring 2007 final exams as a trial run to see if the rubric was easily applied and if the rubric would give us viable data. Therefore, we did not collect final exams from each course, nor did we ask for copies of the students’ work that was being assessed. We simply asked each instructor to use the rubric and provide us with data.

After providing instructors with the rubrics and the instructions for how to conduct the assessments, all instructors provided the necessary data without asking a single question about the selection process or about the rubric. It seems reasonable to conclude that the rubric will be a useful tool in assessing the learning goals of the quantitative reasoning requirement. So, we plan to use this process to assess all quantitative reasoning courses in future semesters. We will have each instructor send a copy of the final exam for each course, copies of the students’ work that was assessed using the rubric, and a summary of their assessments to the Quantitative Reasoning Coordinator.

Analysis of Assessment Results

1. The review of the syllabi for all QR courses revealed that all QR syllabi included topics that directly addressed the QR student learning outcomes. However, most syllabi did not explicitly state the QR learning outcomes. This is not surprising because the learning outcomes were not distributed to QR instructors before they prepared their syllabi.

Table 5. Syllabi Review Rating

Green	Yellow	Red
X		

2. The student enrollment data (Table 2.)reveal the following.

(a) 77.4% of the students in MA112 were education majors. The course was specifically created for these majors.
(b) 64.3% of the students in MA114, MA140, and MA208, collectively, were science and mathematics majors. These courses are designed for these majors.
(c) 37.4% of the students in MA120 were business majors. This course is required of business majors. However, MA120 was also quite popular across the University.
(d) 37.7% of the students in MA125 were fine arts majors. This course also had wide appeal. The course was created to provide QR skills to non-science and non-business majors.
(e) 97.9% of nursing majors chose a statistics course. 60.8% of these chose SO/PS 201 over MA120.
(f) 53.7% of students majoring in areas that we've grouped under "Other" chose a statistics course.

Overall, it seems that students are taking the appropriate QR course.

3. The evaluation of random samples of completed final exams concluded the following status of student performance with respect to the QR student learning outcomes.

(1) use deductive reasoning in a formal, symbolic, axiomatic system.

Table 6. Learning Goal (1) Rating

Green	Yellow	Red
X		

Comment: Nearly 90% of the assessed student completed finals were rated good or average by the instructors.

(2) apply the theorems of the system to solve appropriate problems.

Table 7. Learning Goal (2) Rating

Green	Yellow	Red
X		

Comment: Seventy-one percent of the assessed student completed finals were rated good or average by the instructors. The majority were rated good. The number assessed as poor (29%) is greater that we would like. It would seem that we need to attend to theorem application a little more.

Improvement Plans

We are pleased that the data indicate that the QR goals are being met. Nevertheless, we see room for improvement in several areas.

(1) QR instructors should be instructed to list the QR goals on their syllabi.
(2) The assessment team needs to inform QR instructors early in the semester the team will need copies of their final exams.
(3) The theorem application goal, while being met, seems to need more attention. Chairs should emphasize this to QR instructors.
(4) The next assessment report should include QR courses taught in the PACE program.

Appendix.

Rubric for Assessing Student Achievement of Quantitative Reasoning Learning Outcome Goals

	Good	Average	Poor
Deductive Reasoning	• Student uses proper symbolic notation in the context of the stated problem • Student manipulates these symbols according to the rules of the axiomatic system • Student achieves desired directive of the problem with no (or a few minor) errors	• Student work achieves exactly two of the following: 1. Student uses proper symbolic notation in the context of the stated problem 2. Student manipulates these symbols according to the rules of the axiomatic system 3. Student achieves desired directive of the problem with no (or a few minor) errors	• Student work achieves no more than one of the following: 1. Student uses proper symbolic notation in the context of the stated problem 2. Student manipulates these symbols according to the rules of the axiomatic system 3. Student achieves desired directive of the problem with no (or a few minor) errors
Theorem Application	• Student correctly selects theorem(s) necessary to solve stated problem • Student performs all necessary calculations needed to apply theorem with no (or a few minor) errors • Student uses selected theorem(s) to form a correct conclusion on the basis of the computations	• Student correctly selects theorem(s) necessary to solve stated problem • Student's work falls into one of the following two categories: 1. Student made major computational errors, but made a correct conclusion based on the computations 2. Student made no (or a few minor) errors in computations, but made an incorrect conclusion made on the computations	• Student work falls into one of the following two categories: 1. Student did not correctly select theorem(s) necessary to solve stated problem 2. Student did correctly select theorem(s) necessary to solve, but made major computational errors and made an incorrect conclusion based on the computations

Examples

Deductive Reasoning

MA 120 students are asked to solve the following problem: "On the 2006 SAT Mathematics Exam, the average score was 518 with a standard deviation of 115. Assuming these scores are normally distributed, what percentage of students scored higher than 596 on the 2006 SAT Mathematics Exam?"

This problem requires students to select the *z*-score formula, substitute the given information into the formula, perform the *z*-score computation, find the corresponding percentage in a normal distribution table, and manipulate the number from the table to answer the given question correctly.

The selection of the *z*-score formula and correct substitution into the formula would constitute using proper symbolic notation in the context of the problem because students must extract the necessary information from the problem and replace the unknown quantities in formula with this information. Correctly computing the *z*-score and finding the correct corresponding percentage in the normal distribution table would constitute manipulating these symbols according to the rules of the axiomatic system. Finally, correctly extracting the percentage from the table to answer the given question shows the student not only can follow the rules of the system, but also can reach a valid conclusion after applying these rules, thereby achieving the desired directive.

Good response: *The z-score for an SAT Mathematics score of 596 is*

$$\begin{aligned} z &= \frac{596 - 518}{115} \\ &= \frac{78}{115} \\ &\approx 0.68 \end{aligned}$$

The percentage corresponding to a z-score of 0.68 in the normal distribution table is 25.17%. This is the percentage that lies between the data point and the mean, or in this case, the percentage of students who scored between 518 and 596. So, the percentage of students who scored higher than 596 on the 2006 SAT Mathematics Exam is 50% - 25.17% = 24.83%

Average response: *The z-score for an SAT Mathematics score of 596 is*

$$z = \frac{596 - 518}{115}$$
$$= \frac{78}{115}$$
$$\approx 0.68$$

The percentage corresponding to a z-score of 0.68 in the normal distribution table is 25.17%. So, the percentage of students who scored higher than 596 on the 2006 SAT Mathematics Exam is 25.17%.

This student response demonstrates the first two traits of a good response, but not the third. The student has correctly used proper symbolic notation in the context of the stated problem by selecting the *z*-score formula and inserting the numbers into the formula correctly, and manipulated these symbols according to the rules of the axiomatic system by finding the correct *z*-score and corresponding percentage from the normal distribution table. However, the student has simply used the percentage found in the table as the final answer and thus has not achieved the desired directive of the problem.

Poor response: *The z-score for an SAT Mathematics score of 596 is*

$$z = \frac{596 - 518}{115}$$
$$= \frac{78}{115}$$
$$\approx 0.68$$

So, the percentage of students who scored higher than 596 on the 2006 SAT Mathematics Exam is 68%.

This student response demonstrates the first trait of a good response, but not the other two. The student has correctly selected the *z*-score formula and substitutes the given information correctly. However, the student did not apply the rules of the axiomatic system correctly since the student did not find the corresponding percentage in the normal distribution table. Also, the student did not achieve the desired directive because the student did not apply the rules of the system correctly.

<u>Deductive Reasoning</u>

PH 213 students are asked the following question: "Using a truth table, determine whether the statements $\neg p \vee q$ and $p \rightarrow q$ are equivalent."

This problem requires students to construct a truth table for each of the statements and determine if the statements have identical final columns in their truth tables.

A student uses proper symbolic notation in the context of the stated problem when the student sets up the truth table correctly be creating the appropriate columns for the statements. When the student fills out the truth table correctly, that student has manipulated these symbols according to the rules of the axiomatic system. Finally, the desired directive is achieved when the student compares the final column of each truth table to determine whether or not the statements are equivalent.

Good response:

The truth table for $\neg p \vee q$ *is*

p	q	$\neg p$	$\neg p \vee q$
T	*T*	*F*	*T*
T	*F*	*F*	*F*
F	*T*	*T*	*T*
F	*F*	*T*	*T*

The truth table for $p \rightarrow q$ *is*

p	q	$p \rightarrow q$
T	*T*	*T*
T	*F*	*F*
F	*T*	*T*
F	*F*	*T*

Since the last columns of the truth tables are identical, the statements are equivalent.

The student set up the truth tables correctly, including having an extra column in the first table for $\neg p$ to use as an intermediate step. So, this student has used proper symbolic notation for this problem. This student has also correctly filled in the truth tables, demonstrating the ability to use the rules of the axiomatic system in this context. Finally, the student correctly assesses the statements are in fact equivalent by examining the final columns of the truth table, thereby achieving the desired directive of the problem.

Average response:

The truth table for $\neg p \vee q$ *is*

p	q	$\neg p$	$\neg p \vee q$
T	*T*	*F*	*F*
T	*F*	*F*	*F*
F	*T*	*T*	*T*
F	*F*	*T*	*F*

The truth table for $p \rightarrow q$ is

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Since the last columns of the truth tables are not identical, the statements are not equivalent.

The student set up the truth tables correctly, including having an extra column in the first table for $\neg p$ to use as an intermediate step. So, this student has used proper symbolic notation for this problem. This student has not correctly filled in the truth tables; indeed, this student has confused the logical "or" with the logical "and". So, this student has not followed the rules of the axiomatic system correctly. However, the student does use their information to assess correctly (according to their computations) that the statements are not equivalent by examining the final columns of the truth table, thereby achieving the desired directive of the problem. Since this student work exhibits two of the three criteria of a good response, this response is rated as average.

Poor response:

The truth table for $\neg p \vee q$ is

p	q	$\neg p \vee q$
T	T	T
F	F	F

The truth table for $p \rightarrow q$ is

p	q	$p \rightarrow q$
T	T	T
F	F	T

Since the last columns of the truth tables are not identical, the statements are not equivalent.

The student has not set up the truth tables correctly because the student has eliminated to cases from consideration. So, this student has not used proper symbolic notation for this problem. This student also has not correctly filled in the truth tables, and with the missing rows in the truth tables, it is impossible to determine whether the student has confused the logical "or" with the logical "and", or if the student just randomly inserted some symbols. In either case, this student has not followed the rules of the axiomatic system correctly. However, the student does use their information to assess correctly (according to their computations) that the statements are not equivalent by examining the final columns of the truth table, thereby achieving the desired directive of the problem. Since this student work exhibits one of the three criteria of a good response, this response is rated as poor.

Theorem Application

MA 140 students are asked the following question: "For the function $f(x) = x^3 - 6x^2$, find the *x*-coordinates of the points at which *f(x)* has relative maxima and relative minima."

This problem requires students to find the derivative of *f(x)*, denoted $f'(x)$, to solve the equation $f'(x) = 0$ and to determine when $f'(x)$ is undefined to identify the critical numbers of *f(x)*, and to apply either the First Derivative Test or the Second Derivative Test to each of the critical numbers in order to identify which, if any, of the critical numbers correspond to relative maxima and which, if any, correspond to relative minima.

The computation of the derivative and solving $f'(x) = 0$ would qualify as performing all necessary calculations needed to apply the theorem. Students can show they correctly select the theorem necessary to solve stated problem by applying either the First Derivative Test or the Second Derivative Test. If students apply the theorem correctly, they will correctly identify where the maxima and minima are and thus demonstrate the third criterion of the rubric.

Good response:

$$f(x) = x^3 - 6x^2$$

$$f'(x) = 3x^2 - 12x$$

We see that $f'(x)$ *is undefined nowhere and that* $f'(x) = 0$ *when*

$$3x^2 - 12x = 0$$

$$3x(x-4) = 0$$

$$x = 0 \text{ or } x = 4$$

Now, $f''(x) = 6x - 12$. *Since* $f''(0) = -12 < 0$, *the Second Derivative Test tells us that* $x = 0$ *is the x-coordinate of a relative maximum of f(x). Since* $f''(4) = 12 > 0$, *the Second Derivative Test tells us that* $x = 4$ *is the x-coordinate of a relative minimum of f(x). So, the function* $f(x) = x^3 - 6x^2$ *has a relative maximum at* $x = 0$ *and a relative minimum at* $x = 4$.

This student response correctly computes the derivative and finds all the critical number. So, the second trait of a good response has been met. The first trait, the correct selection of the theorem, is shown by the student correctly selecting the Second Derivative Test to use. Finally, the student applies the theorem correctly by plugging the critical numbers into the second derivative and makes the correct conclusions based on these computations. Thus, the third trait of the good response has been met.

Average response:

$$f(x) = x^3 - 6x^2$$

$$f'(x) = 3x^2 - 12x$$

We see that $f'(x)$ *is undefined nowhere and that* $f'(x) = 0$ *when*

$$3x^2 - 12x = 0$$

$$3x(x-4) = 0$$

$$x = 0 \text{ or } x = 4$$

Now, $f''(x) = 6x - 12$. *Since* $f''(0) = -12 < 0$, *the Second Derivative Test tells us that* $x = 0$ *is the x-coordinate of a relative minimum of f(x). Since* $f''(4) = 12 > 0$, *the Second Derivative Test tells us that* $x = 4$ *is the x-coordinate of a relative maximum of f(x). So, the function* $f(x) = x^3 - 6x^2$ *has a relative minimum at* $x = 0$ *and a relative maximum at* $x = 4$.

This student response correctly computes the derivative and finds all the critical number. So, the second trait of a good response has been met. The first trait, the correct selection of the theorem, is shown by the student correctly selecting the Second Derivative Test to use. The student applies the theorem correctly by plugging the critical numbers into the second derivative; however, the student makes incorrect conclusions by saying the maximum is a minimum, and vice versa. Thus, the third trait of the good response has not been met.

Poor response:

$$f(x) = x^3 - 6x^2$$

$$f'(x) = 3x^2 - 12x$$

We see that $f'(x)$ *is undefined nowhere and that* $f'(x) = 0$ *when*

$$3x^2 - 12x = 0$$

$$3x(x-4) = 0$$

$$x = 0 \text{ or } x = 4$$

Now, $f(0) = 0$ *and* $f(4) = -32$. *So, f(x) has a relative minimum at* $x = 4$ *and a relative maximum at* $x = 0$.

This student has performed correct calculations when finding the derivative and the critical numbers. However, the student has not applied the appropriate theorem to arrive at a correct conclusion. In fact, this student has attempted to use the Extreme Value Theorem, which deals with the *absolute* maximum and *absolute* minimum of a function under certain conditions. This is a major conceptual error and thus warrants a rating of poor.